Department of Applied Mathematics
PROBABILITY AND STATISTICS PRELIMINARY EXAMINATION
August 2018

Instructions:

Do two of three problems in each section (Stat and Prob).
Place an X on the lines next to the problem numbers
that you are NOT submitting for grading.

Prob	
1.	____
2.	____
3.	____

Please do not write your name anywhere on this exam.
You will be identified only by your student number.
Write this number on each page submitted for grading.
Show all relevant work.

Stat	
4.	____
5.	____
6.	____
Total	____

Student Number ____________________________

Probability Section

1. Probability: Problem 1

Let c R be a constant, and consider a random vector (X, Y) taking values in R^2 with probability density function:

$$f(x, y) = \frac{1}{2\pi} \exp \frac{2cxy - (1 + c^2)x^2 - y^2}{2} .$$

(a) Determine the distribution of X.

(a) Use (1) and $e^x = \ {}_{n=0}^{\infty} x^n/n!$ to show that $P(A) = 1 + q_{ii}\varepsilon + \frac{1}{2}q_{ii}^2\varepsilon^2 + o(\varepsilon^2)$. From this, prove that $E \ {}_0^{\varepsilon} f(X_t)dt \quad A \ \ P(A) = f(i)\varepsilon + q_{ii} f(i)\varepsilon^2 + o(\varepsilon^2)$.

(b) Find the conditional density function of τ given that $\tau \le \varepsilon$.
(Hint: You may first derive the conditional distribution of τ given that $\tau \le \varepsilon$).

(c) Let $\tau \ := \inf\{t \ge \tau : X_t = X_\tau\}$, the second time X changes its state. Since $B = \{\tau \le \varepsilon < \tau \}$, the quantity $E \ {}_0^{\varepsilon} f(X_t)dt$